

Statistics S2 Mark scheme

Question	Scheme	Marks
1(a)	$X \sim \text{Po}(3.2)$	B1
	$P(X = 3) = \frac{e^{-3.2} 3.2^3}{3!}$	M1
	$= 0.2226$ awrt 0.223	A1
		(3)
(b)	$Y \sim \text{Po}(1.6)$	B1
	$P(Y \geq 1) = 1 - P(Y = 0)$ $= 1 - e^{-1.6}$	M1
	$= 0.7981$ awrt 0.798	A1
		(3)
(c)	$X \sim \text{Po}(0.8)$	
	$\frac{P(X = 1) \times P(X = 3)}{P(Y = 4)} = \frac{(e^{-0.8} \times 0.8) \times \left(\frac{e^{-0.8} 0.8^3}{3!} \right)}{\frac{e^{-1.6} 1.6^4}{4!}}$ $= \frac{0.3594 \times 0.0383}{0.05513}$	M1 M1 M1 A1
	$= 0.25$	A1
		(5)
(d)	$A \sim \text{Po}(72)$ approximated by $N(72, 72)$	B1
	$\frac{5000}{60} = 83.33$	M1
	$P(A \geq 84) = P\left(Z \geq \frac{83.5 - 72}{\sqrt{72}}\right)$	M1 M1
	$= P(Z \geq 1.355\dots)$ $= 0.0869$ awrt 0.087/0.088	A1
		(5)
		(16 marks)
Notes:		
(a)		
B1: For writing or using $\text{Po}(3.2)$		
M1: $\frac{e^{-\lambda} \lambda^3}{3!}$		
(b)		
B1: For writing or using $\text{Po}(1.6)$		
M1: $1 - P(Y = 0)$ or $1 - e^{-\lambda}$		

Question 1 notes *continued*

(c)

M1: Using Po(0.8) with $X=1$ or $X=3$ (may be implied by 0.359... or 0.0383...)

M1: $(e^{-\lambda} \times \lambda) \times \left(\frac{e^{-\lambda} \lambda^3}{3!} \right)$ (consistent lambda) awrt 0.0138 implies 1st 2 M marks

M1: Correct use of conditional probability with denominator = $\frac{e^{-1.6} 1.6^4}{4!}$

A1: Fully correct expression

A1: 0.25 (allow awrt 0.250)

(d)

B1: Writing or using N(72,72)

M1: For exact fraction **or** awrt 83.3 (may be implied by 84)
(Note: Use of N(4320,4320) can score B1 and 1st M1)

M1: Using 84 +/- 0.5

M1: Standardising using 82.5, 83, 83.3̇ (awrt 83.3), 83.5, 83.8, 84 or 84.5, 'their mean' **and** 'their sd'

Question	Scheme	Marks
2(a)	$P(X > 4) = 1 - F(4)$	M1
	$= 1 - \frac{3}{5}$	
	$= \frac{2}{5}$ oe	A1
		(2)
(b)	1	B1
		(1)
(c)	$f(x) = \frac{dF(x)}{dx} = \frac{1}{5}$	M1
	$f(x) = \begin{cases} \frac{1}{5} & 1 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$	A1
		(2)
(d)	$E(X) = 3.5$	B1
		(1)
(e)	Variance = $\frac{(6-1)^2}{12}$ or $\int_1^6 \frac{1}{5} x^2 dx - (3.5)^2$	M1
	$= \frac{25}{12}$ awrt 2.08	A1
		(2)
(f)	$E(X^2) = \text{Var}(X) + [E(X)]^2$	
	$= \frac{25}{12} + 3.5^2$ or $\int_1^6 \frac{1}{5} x^2 dx$ or $\int_1^6 \frac{1}{5} (3x^2 + 1) dx$	M1
	$= \frac{43}{3}$	
	$E(3X^2 + 1) = 3 E(X^2) + 1$	dM1
	$= \left[\frac{3x^3}{15} + \frac{x}{5} \right]_1^6$	
	$= 44$	A1cao
		(3)
(11 marks)		
Notes:		
(a)		
M1: Writing or using $1 - F(4)$ o.e.		
(c)		
M1: For differentiating to get $\frac{1}{5}$		

Question 2 notes *continued*

A1: Both lines correct with ranges

(e)

M1: $\frac{(6-1)^2}{12}$ or $\int_1^6 \frac{1}{5} x^2 dx$ – ‘their 3.5’²

(f)

M1: “Their $\text{Var}(X)$ ” + [“their $E(X)$ ”]² (which must follow from the 1st method in (e))

or $\int_1^6 \frac{1}{5} x^2 dx$ **and** integrating $x^n \rightarrow \frac{x^{n+1}}{n+1}$ (may be seen in (e)) **or** writing $\int_1^6 \frac{1}{5} (3x^2 + 1) dx$

(May be implied by $\frac{43}{3}$ seen)

dM1: Using $3 \times$ ‘their $E(X^2)$ ’ + 1 **or** $\int_1^6 \frac{1}{5} (3x^2 + 1) dx$ and integrating $x^n \rightarrow \frac{x^{n+1}}{n+1}$

Question	Scheme	Marks	
3(a)	(A random variable) that is a function of a (random) sample involving no unknown quantities/parameters	B1	
	or A quantity calculated solely from a random sample		
		(1)	
(b)	If all possible samples are chosen from a population;	B1	
	then the values of a statistic and the associated probabilities is a sampling distribution		
	or a probability distribution of a statistic		
		(1)	
(c)	Mean = $100 \times \frac{4}{7} + 200 \times \frac{3}{7}$	B1	
	= $\frac{1000}{7}$ awrt 143		
	Variance = $100^2 \times \frac{4}{7} + 200^2 \times \frac{3}{7} - \left(\frac{1000}{7}\right)^2$		M1
	= $\frac{120000}{49}$ awrt 2450 (to 3sf)		A1
		(3)	
(d)	(100,100,100)	B2	
	(100,100,200) (100,200,100) (200,100,100) or 3 x (100,100,200)		
	(100,200,200) (200,100,200) (200,200,100) or 3 x (100,200,200)		
	(200,200,200)		
		(2)	
(e)	(100,100,100) $\left(\frac{4}{7}\right)^3 = \frac{64}{343}$ awrt 0.187	B1 both	
	(200,200,200) $\left(\frac{3}{7}\right)^3 = \frac{27}{343}$ awrt 0.0787		
	(100,100,200) $3 \times \left(\frac{4}{7}\right)^2 \times \left(\frac{3}{7}\right) = \frac{144}{343}$ awrt 0.420 (allow 0.42)		M1
	(100,200,200) $3 \times \left(\frac{4}{7}\right) \times \left(\frac{3}{7}\right)^2 = \frac{108}{343}$ awrt 0.315		A1

Question	Scheme					Marks
3(e) <i>continued</i>	m	100	$\frac{400}{3}$ awrt 133	$\frac{500}{3}$ awrt 167	200	A1
	$P(M = m)$	$\frac{64}{343}$ or awrt 0.187	$\frac{144}{343}$ or awrt 0.420 (allow 0.42)	$\frac{108}{343}$ or awrt 0.315	$\frac{27}{343}$ or awrt 0.0787	
						(4)
(11 marks)						
Notes:						
<p>(a)</p> <p>B1: For a definition which includes each of the following 3 aspects A function¹ of a (random) sample² involving no unknown quantities/parameters³</p> <ol style="list-style-type: none"> 1. function/quantity/calculation/value/random variable 2. sample/observations/data 3. no unknown parameters/no unknown values/solely (from a sample) 						
<p>(b)</p> <p>B1: Requires all underlined words: <u>All values</u> of a <u>statistic</u> with their associated <u>probabilities</u> or <u>probability distribution</u> of a <u>statistic</u></p>						
<p>(c)</p> <p>M1: $100^2 \times \frac{4}{7} + 200^2 \times \frac{3}{7} - (\text{their mean})^2$</p>						
<p>(d)</p> <p>B1: Any 2 of (100,100,100), (100,100,200) any order, (100,200,200) any order, (200,200,200)</p> <p>B1: All correct, allow $3 \times (100,100,200)$ and $3 \times (100,200,200)$ and (100,100,100) and (200,200,200)</p> <p>(Note: Allow other notation for 100 and 200 e.g. Small and Large)</p>						
<p>(e)</p> <p>B1: Both probabilities for (100,100,100) and (200,200,200) correct</p> <p>M1: $3 \times p^2 \times (1 - p)$</p> <p>A1: Either correct</p> <p>A1: All means correct and all probabilities correct (table not required but means must be associated with correct probabilities)</p>						

Question	Scheme	Marks
4(a)	$X \sim \text{Po}(6)$	M1
	$P(5 \leq X < 7) = P(X \leq 6) - P(X \leq 4)$ or $\frac{e^{-6}6^5}{5!} + \frac{e^{-6}6^6}{6!}$	M1
	$= 0.6063 - 0.2851$	
	$= 0.3212$ awrt 0.321	A1
		(3)
(b)	$H_0: \lambda = 9$ $H_1: \lambda < 9$	B1
	$X \sim \text{Po}(9)$ therefore $P(X \leq 4) = 0.05496\dots$ or CR $X \leq 3$	B1
	Insufficient evidence to reject H_0 or Not Significant or 4 does not lie in the critical region.	dM1
	There is no evidence that the mean number of <u>accidents</u> at the crossroads has <u>reduced/decreased</u> .	A1cso
		(4)
(7 marks)		
Notes:		
(a)		
M1: Writing or using $\text{Po}(6)$		
M1: Either $P(X \leq 6) - P(X \leq 4)$ or $\frac{e^{-\lambda}\lambda^5}{5!} + \frac{e^{-\lambda}\lambda^6}{6!}$		
(b)		
B1: Both hypotheses correct (λ or μ) allow 0.5 instead of 9		
B1: Either awrt 0.055 or critical region $X \leq 3$		
dM1: For a correct comment (dependent on previous B1)		
Contradictory non-contextual statements such as “not significant” so “reject H_0 ” score M0. (May be implied by a correct contextual statement)		
A1: Cso requires correct contextual conclusion with underlined words and all previous marks in (b) to be scored.		

Question	Scheme	Marks
5(a)	$\int_{-1}^2 k(x^2 + a)dx + \int_2^3 3k dx = 1$	M1
	$\left[k \left(\frac{x^3}{3} + ax \right) \right]_{-1}^2 + [3kx]_2^3 = 1$	dM1
	$k \left(\frac{8}{3} + 2a + \frac{1}{3} + a \right) + 9k - 6k = 1$	A1
	$6k + 3ak = 1$ $\int_{-1}^2 k(x^3 + ax)dx + \int_2^3 3kx dx \left[= \frac{17}{12} \right]$	M1
	$\left[k \left(\frac{x^4}{4} + \frac{ax^2}{2} \right) \right]_{-1}^2 + \left[\frac{3kx^2}{2} \right]_2^3 = \frac{17}{12}$	dM1
	$k \left(4 + 2a - \frac{1}{4} - \frac{a}{2} \right) + \frac{27k}{2} - 6k = \frac{17}{12}$	A1
	$\frac{45k}{4} + \frac{3ak}{2} = \frac{17}{12}$ $135k + 18ak = 17$ $99k = 11$	ddM1
	$a = 1, k = \frac{1}{9}$	A1
		(8)
(b)	2	B1
		(1)

(9 marks)

Notes:

(a)

M1: Writing or using $\int_{-1}^2 k(x^2 + a)dx + \int_2^3 3k dx = 1$ ignore limits.

dM1: Attempting to integrate at least one $x^n \rightarrow \frac{x^{n+1}}{n+1}$ **and** sight of correct limits (dependent on previous M1).

A1: Correct equation – need not be simplified.

M1: $\int_{-1}^2 k(x^3 + ax)dx + \int_2^3 3kx dx$ ignore limits.

dM1: Setting $= \frac{17}{12}$ **and** attempting to integrate at least one $x^n \rightarrow \frac{x^{n+1}}{n+1}$ **and** sight of correct limits (dependent on previous M1).

Question 5 notes *continued*

A1: A correct equation – need not be simplified.

ddM1: Attempting to solve two simultaneous equations in a and k by eliminating 1 variable (dependent on 1st and 3rd M1s).

A1: Both a and k correct.

Question	Scheme	Marks
6(a)	$P(X = 5) = {}^{20}C_5(0.3)^5(0.7)^{15}$ or $0.4164 - 0.2375$	M1
	$= 0.17886\dots$ awrt 0.179	A1
		(2)
(b)	Mean = 6	B1
	sd = $\sqrt{20 \times 0.7 \times 0.3}$	M1
	$= 2.049\dots$ awrt 2.05	A1
		(3)
(c)	$H_0: p = 0.3$ $H_1: p > 0.3$	B1
	$X \sim B(20, 0.3)$	M1
	$P(X \geq 8) = 0.2277$ or $P(X \geq 10) = 0.0480$, so CR $X \geq 10$	A1
	Insufficient evidence to reject H_0 or Not Significant or 8 does not lie in the critical region.	dM1
	There is no evidence to support the <u>Director (of Studies') belief/</u> There is no evidence that the <u>proportion of parents that do not support the new curriculum</u> is greater than 30%	A1 cso
	(5)	
(d)	$X \sim B(2n, 0.25)$	
	$X \sim B(8, 0.25)$ $P(X \geq 4) = 0.1138$	M1
	$X \sim B(10, 0.25)$ $P(X \geq 5) = 0.0781$	
	$2n = 10$	A1
	$n = 5$	A1
	(3)	
(13 marks)		
Notes:		
(a)		
M1: ${}^{20}C_5(p)^5(1-p)^{15}$ or using $P(X \leq 5) - P(X \leq 4)$		
(b)		
M1: Use of $20 \times 0.7 \times 0.3$ (with or without the square root).		
(c)		
B1: Both hypotheses correct (p or π).		
M1: Using $X \sim B(20, 0.3)$ (may be implied by 0.7723, 0.2277, 0.8867 or 0.1133)		
A1: Awrt 0.228 or CR $X \geq 10$		
dM1: A correct comment (dependent on previous M1)		
A1: Cso requires correct contextual conclusion with underlined words and all previous marks in (c) to be scored.		

Question 6 notes *continued*

(d)

M1: For 0.1138 or 0.0781 or 0.8862 or 0.9219 seen.

A1: B(10, 0.25) selected (may be implied by $n = 10$ or $2n = 10$ or $n = 5$).

An answer of 5 with no incorrect working seen scores 3 out of 3.

Special Case: Use of a normal approximation.

M1: For $\frac{(n-0.5) - \frac{n}{2}}{\sqrt{\frac{3}{8}n}} = z$ with $1.28 \leq z \leq 1.29$, 1st A1 for $n=4.2/4.3$, 2nd A1 for $n=5$

Question	Scheme	Marks
7	$Y \sim N\left(\frac{n}{5}, \frac{4n}{25}\right)$	B1
	$P(Y \geq 30) = P\left(Z > \frac{29.5 - n/5}{\frac{2}{5}\sqrt{n}}\right)$	M1 M1 A1
	$\frac{29.5 - n/5}{\frac{2}{5}\sqrt{n}} = 2$	B1
	$n + 4\sqrt{n} - 147.5 = 0$ or $0.04n^2 - 12.44n + 870.25 = 0$	dM1
	$\sqrt{n} = 10.3\dots$ or $n = 106.26\dots$ or $n = 204.73\dots$	A1
	$n = 106$	A1 cao
(8 marks)		
Notes:		
<p>B1: Writing or using $N\left(\frac{n}{5}, \frac{4n}{25}\right)$</p> <p>M1: Writing or using 30 ± 0.5</p> <p>M1: Standardising using 29, 29.5, 30 or 30.5 and their mean and their sd</p> <p>A1: Fully correct standardisation (allow +/-)</p> <p>B1: For $z = \pm 2$ or awrt 2.00 must be compatible with their standardisation</p> <p>dM1: (Dependent on 2nd M1) getting quadratic equation and solving leading to a value of \sqrt{n} or n</p> <p>A1: Awrt 10.3 or awrt (106 or 107 or 204 or 205)</p> <p>A1: For 106 only (must reject other solutions if stated)</p> <p>(Note: $\frac{29.5 - n/5}{\frac{2}{5}\sqrt{n}} = -2$ leading to an answer of 106 may score B1M1M1A1B0M1A1A1)</p>		